Notes for a Course on Financial Mathematics

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To the Student

This is not the first time these problems have been used. However, this is only the second time they have been used in this typed version. As a result, I know that there are typos. When you find typos, please let me know so that I can fix them in future versions of these notes, and definitely let me know when you find impossible problems (there might be a few) - these might be problems lacking enough information or with dates reversed or some such thing. Think critically about the problems before reporting them as impossible, but feel free to ask questions (in person or via email) when you notice discrepancies.

0.1 How to Use This Book

Each topic in this book has two parts - problems to be completed in class (either as part of lecture or as worksheets) and homework problems. We will discuss each of these in kind.

The in-class portion for each topic includes definitions and formulas to help you understand new ideas. The part of the book that usually includes examples of how to complete problems to demonstrate a certain idea are the ones that you will be completing in class. This is much like getting the power point slides for a lecture and having to fill in the details. However, here the details are the calculations to complete different types of problems. You should find a way to keep yourself organized so that you can figure out later how your notes fit in with the handouts from class.

You will notice that all problems (in-class exercises and homework) are consecutively numbered, so you might want to have just one notebook that numbers all problems that you complete over the course of the semester.

The homework portion of the topic are problems that you should complete at home (hence the name “homework”). These problems are to give you extra practice with completing the types of problems that we have discussed in class. I hope that you will have opportunity to practice many of the problems in class, but homework problems are NECESSARY. Notice that the homework sets are not hundreds of problems, where the instructor assigns a few select problems. These ARE the few select problems. There-
fore, you are expected to complete ALL of the homework problems. You will be responsible for these problems (and ones like them) on quizzes and exams. Therefore, you should ask questions about problems that you are not able to complete.

Many of you probably felt like running away when you realized that you had to take a mathematics class. I hope that I can convince you that you can, instead, find much interesting and helpful information in this course and should instead feel calm about it - and maybe even happy!

A wise man once said that learning mathematics is like learning to play the piano - you can see how it is done by watching, but you only really learn the skills by practicing them yourself. You will know if you are not practicing enough because your quiz scores will be low - this means that you need to do more homework, make sure that you are in class all the time, and ask questions when necessary. You may have heard the old expression “You can lead the horse to water, but you can’t make it drink”? It is my job to lead you to water, and your job to drink. Good luck!

Jacqueline A. Jensen-Vallin

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Chapter 1

Basic Definitions and Simple Interest

1.1 Basic Definitions

Definition 1. **Principal** is the money that is borrowed or invested.

Definition 2. **Interest** is the cost or charge for borrowing money. This is also known as the interest earned, the return on investment, dividend, and coupon for investment accounts.

Definition 3. The **interest rate** is the ratio of the interest charged for a loan to the principal for a given time period, usually annually (yearly).

Definition 4. A **simple interest loan** is a loan for which interest is just based on the original principal but paid at the end of the term.

Formula 1. For a simple interest loan

\[ I = Prt \]

where \( I \) is the interest earned, \( P \) is the principal invested, \( r \) is the interest rate, and \( t \) is the time in years for which the money will be invested.

Use the definitions and formula above to answer each of the following:

1. Find interest on $1000 at 8.2% annually for 8 months.

2. Find the interest on $2950.50 at 7.75% annually for 20 months.

3. On 11/2/10 Marge invests $5000 at 4.5%. Find the interest and the future value of the investment 9 months later.

Definition 5. The **future value** (a.k.a., maturity value or amount) of an investment is the total the investment is worth (or the total amount the debt will be worth) at the term of the loan. This is equal to principal plus interest.
Formula 2. *Future value is given by the formula*

\[ S = P + I \]

Formula 3. *For a simple interest loan, the future value of the loan is*

\[ S = P + Prt = P(1 + rt) \]

4. Homer requires a loan of $125,000 to start an internet business. He found a loan agent that charges 14% interest. He is not willing to pay more than $1000 per month. Why is this an impossible situation?

5. On 1/2/03, Patty lends her sister, Selma, $1000. Selma pays off the loan on 7/2/03 with $1050. Find the annual interest rate.

6. If you hope to double your money and you have $1000 to invest at 6%, how long will it take you?

1.1.1 HW problems for simple interest problems

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

7. Find the interest paid when $400 is deposited at 6.5% for 8 months.

8. Find the interest paid when $325.80 is borrowed at 18% for 14 months.

9. Find the interest paid when $150,000 is borrowed at 4.3% for 14 weeks.

10. Find the interest paid when $8500 is borrowed at 3.25% for 40 weeks.

11. On 11/2/02, Lisa invests $1000 at 4.5%. Find the interest and the future value of the investment on 4/2/04.

12. On 9/5/03, Bart takes out a loan for $1000 from his parents, who are charging 7%. When Bart enters middle school on 9/5/06, how much interest does he owe, and what is the amount he will repay his parents?

13. Edna requires a quick loan of $5000 and uses a credit card that charges 19%. She can pay at most $60 per month. Can she ever repay the loan? Why or why not?

14. A quick check-cashing agent is willing to hold your check of $500 for 2 weeks for only $30. Find the annual interest rate.

15. To the nearest month, how long does it take to increase your investment by 50% if you can get 4% on your investment?

16. If you desire to triple your money at 10%, how long will it take you?
1.2 Exact and Approximate Time (and Interest)

**Definition 6.** When using **exact time**, interest is computed on the exact number of days the money is borrowed for (except for the first day).

17. Start on January 1st, numbering that day 1, and write out the numbers for all of the days through March 30th.

18. List each of the twelve months and the number of days in each.

19. Find the exact time for a loan taken out on January 3rd and repaid on September 3rd. (Question - is this always the same number of days?!?!) 

20. Find the exact time for a loan taken out on February 14, 2008 and repaid on October 15, 2008.

21. How can you tell if a year is a leap year without having to look at a calendar?

22. Are there any exceptions to the rule that you found above?

23. How many days are there in a year using exact time?

**Definition 7.** When using **approximate time**, assume that every month has 30 days. Count the number of full months in the loan period, and multiply them by 30. Then add in the extra days included in the loan period using exact time.

24. Find the approximate time from January 3rd to September 3rd. (Question - is this always the same number of days?!?!) 

25. Find the approximate time from February 14th to October 15th. (Question - is this always the same number of days?!?!) 

26. How many days are there in a year using approximate time?

27. Find the interest on a loan of $4500 at 6% borrowed on August 30, 2010 to be repaid on January 15, 2011.

   (a) Use exact time.

   (b) Use approximate time.

   (c) Are these the same?

   (d) Which would you rather the bank use and why?

**Definition 8.** **Exact interest** calculates interest by assuming a total of 365 days in a year. (This is the denominator of your fraction of a year when calculating time.)
Definition 9. **Ordinary interest** calculates interest by assuming a total of 360 days in a year. (This is the denominator of your fraction of a year when calculating time.)

Definition 10. **Banker’s Rule** says that you calculate the interest on the loan using exact time, but with ordinary interest (assuming 360 days per year). In other words you calculate the length of the loan using exact time, but using approximate time to calculate the number of days in a year).

28. Joe takes a loan of $1000 at 5% on January 1, 2011 to be repaid on January 1, 2012. Find the amount of interest on the loan:
   (a) using exact time and exact interest.
   (b) using approximate time and ordinary interest.
   (c) using Banker’s Rule.
   (d) Which of these charges the most interest and why?

If no other information is provided, **always assume Banker’s Rule**.

29. A 182-day $1,000,000 Treasury bill is bought with a bid of 96.2%. Find the rate of return (i.e., the interest rate the investor earns.).

### 1.2.1 HW for Exact and Approximate Interest

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

30. Was 2008 a leap year? Why or why not?
31. Was 2000 a leap year? Why or why not?
32. Calculate the exact time and approximate time from January 31, 2004 to October 25, 2004.
33. Calculate the exact time and approximate time from September 4, 2004 to October 25, 2004.
34. Find the amount of interest on $2500 at 7.5% between March 5, 2010 and June 5, 2010 using:
   (a) exact time and exact interest
   (b) approximate time and exact interest
   (c) approximate time and ordinary interest
(d) Banker’s Rule

35. On June 6th, Helen borrows $14,000 for new pews at 10%. How much is the interest and what is the payoff on August 15? (Does it matter whether or not this is a leap year?)

36. A 273-day $1,000,000 Treasury bill (T-bill) is bought with a bid of 98.32%. Find the rate of return (ROR), which is the interest rate the investor earns. Notice that for a T-bill the face value is the future value and the bid price is the present value.

37. Find the ROR for a 90-day T-bill bought with a bid of 95% of the face value.

38. You borrow $1200 on December 3, 2007. If interest is 5.5%, find the amount that you repay on April 23, 2008.

39. On 4/12/04, Barney has $3000 in his savings account, which earns only 2.75%. The next interest date is 10/1/04. If Barney withdraws his money early, he will lose all of the interest on the part withdrawn. On 8/15/04, he needs $1500 to pay for astronaut lessons. He finds a bank that will loan him the money at 9%. Should he get the loan or should he withdraw the money from his savings account and why?
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1.3 Future Value and Discount

Definition 11. The future value (a.k.a., the future amount or maturity value) is the value of money at some future date due to the addition of interest.

Remember that future value is 

\[ S = P + I = P(1 + rt). \]

40. What is the future value of $3000 2 years from now at 7%?

41. Seymour owes $1500 on 12/10/03 and $6000 on 3/15/2004. Find the single value on 7/21/04 that he must pay to settle both accounts if money is worth 10% simple interest.

Definition 12. Prompt payment discounts are discounts, as a percent of the cost of the item, that are sometimes given if you pay quickly.

42. Clancey buys a new air conditioner for $5520. The invoice terms are 2/10, n/45. (This means he gets a 2% discount if he pays it by the 10th day and that the net is due in 45 days.) What rate of interest will he earn if he makes his payment on the 10th day?

43. Ned’s Leftorium buys $5520 worth of left-handed brushes and paint, and receives an invoice with terms 3/5, n/60. If Ned pays on the 5th day, what rate of interest will he earn?

44. Buy goods worth $59,865 at terms 2/10, n/45. Repay on the 8th day. Find the interest rate earned.

1.3.1 HW problems for FV and Discount

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

45. How much is $1000 worth 6 months from now at 8%?

46. How much is $2000 worth 8 months from now at 12%?

47. Find the future value of $540 that is due in 18 months at 6%.

48. Martin owes $450 on 12/5/2010 and $550 on 3/1/2011. Find the single value on 7/1/2011 that he paid to discharge both amounts if money is worth 6%.
49. Montie owes $25,000,000 on 12/5/2010 and $15,230,000 on 6/2/2010. Find the single amount he must pay on 7/15/2011 to repay both amounts if money is worth 9% simple interest.


51. Reverend Lovejoy buys a new $10,000 pipe organ for his church and receives an invoice from Springfield Music with terms 3/10, n/60. If he pays on the 10th day, what rate of interest will he earn?

52. The Shelbyville Trust Company pays 5% on Money Market savings accounts, and deposits by the 10th earn interest from the 1st of the month. Interest is credited on June 30 and December 31 each year. Agnes’ account had a balance of $1252.75 on December 31, 2008. She deposited $234.50 on February 8, 2009, and $175.75 on April 4, 2009. If no other deposits were made, what is her balance on June 30, 2009? Use Banker’s Rule.

53. A note for $2250 is due on March 1, 2009 and another for $3500 on June 1, 2009. If money is worth 7%, what single payment on August 1, 2009 will discharge both of these obligations?

1.4 Present Value of Simple Interest

Definition 13. The present value of an investment is the amount of money that would have to be invested today in order to have some given future value at a certain amount of time later.

Formula 4. We already had the future value formula for simple interest given by \( S = P(1 + rt) \). Just solving this for the present value, \( P \), we get the new present value formula:

\[
P = \frac{S}{1 + rt}
\]

54. The Shelbyville Bank and Trust is charging 12.6% for an unsecured loan. Troy owes $21,000 on December 1, 2004, but decides to pay the loan early on February 1, 2004. Find the payoff amount.

55. Find the payoff on 1/1/03 of $1000 due 7/1/03 and $2500 due 3/1/05 at 6%. (Notice that this is a two step problem!)

56. Is it better to pay $3000 in cash for a used car or wait 6 months and pay $3200 if money can earn 18%?
57. Homer needs to pay for parts to repair his plow. Assuming he can earn 18% on his money, should he pay $2000 in cash or wait a year and pay $2200?

58. What investment now will provide the resources in 15 months to pay $8525, if money is worth 6.5%?

59. Springfield Bank and Trust paid off a reserve requirement loan on 8/16/04 with an electronic transfer of $7,849,987.43. How much did they borrow on 7/21/04 at 2.5% and what were the interest charges? Use exact time and exact interest.

### 1.4.1 HW problems for PV of Simple Interest

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

60. The Ogdenville State Bank is charging 6% on an unsecured loan. Doris owes $11,000 on 12/1/07. How much does she repay on 1/1/08?

61. The Ogdenville State Bank is charging 6% on an unsecured loan. Doris owes $11,000 on 12/1/07, but decides to pay off the loan early on 1/1/07. Find the payoff.

62. Homer has incurred debts of $3000 due on 8/1/10 for a plasma TV and $2000 due on 10/2/10 for a new refrigerator. On 6/15/10, Homer has enough cash from worker’s compensation to pay off both debts. Find the payoff at 6.5%.

63. Suppose that Homer is late on the first debt, but pays off both debts on 8/15/10. Find the payoff.

64. Find the cash equivalent at 8% on 1/1/09 of $1500 due on 7/1/09 and $2350 due on 3/1/10.

65. If money costs 8% is it better to pay $3000 in cash or wait 6 months and pay $3200? What is the cash equivalent of the savings resulting from the better buy? Explain your answer.

66. Milhouse’s dad needs parts to repair his boat. If he is earning 20% on his money, should he pay $2000 in cash or wait a year and pay $2500?

67. What should you pay to buy a security that matures to $50,000 in 7 months if you wish to earn 12% on the investment?

68. If a capital expenditure costs $27,500 on 9/6/10 and $32,500 on 1/3/11, what is the better date to make the purchase if money is worth 8%?
69. A bank heist allowed Sideshow Bob to pay off two debts early. What did he pay on 3/15/08 to discharge a $5500 note due on 4/17/08 and a $7650 note due on 8/15/08 if the interest rate is 6.25%?

70. The Springfield Nuclear Power Plant intends to make two equal deposits on 5/1/09 and 8/15/09 to provide for a capital expenditure of $765,890 on 10/18/09. Find the value of those deposits if money is worth 3%.

1.5  Simple Interest Debt Instruments

Sometimes notes are sold to third parties - the investor pays less than the maturity value and gets that amount at maturity date.

71. On February 1, 2003, Homer bought a $3000 120-day note. On March 15, 2003, he needs cash and sells his note to Barney, who requires 8% on his investment. How much did Barney pay for the note?

72. Homer buys a $1000 note that pays 7% interest on June 15, 2004 that matures on November 20, 2004. He sells the note to Moe on October 1, 2004. Moe needs to make 9% on his investment. How much does Moe pay for the note?

73. A $4000 corporate note paying 5% interest is bought on 3/5/04 and matures on 9/5/04. On 7/5/04 this note is sold to a third party requiring 7%.

(a) Find the amount the third party paid for the bond.
(b) What interest did the original owner make?
(c) What interest rate did they make?
(d) Why would he sell?

74. A tax-free municipal security is bought on 3/5/05 and matures to $40,000 on 9/5/05. On 7/5/05 this security is sold to a third party requiring 10%.

(a) Find the maturity time.
(b) Find the future value.
(c) How much time did the second investor hold the investment?
(d) How much did the second investor pay for the investment?
(e) How much interest did the original owner lose?

75. On 1/1/04 the city of Shelbyville issues a tax free $2000 municipal security that matures on 4/10/04 with interest at 8%. If the owner of a bond sold it on 2/1/04 for $2020, what rate of return did the buyer get?
1.5.1 Homework problems for Debt Instruments

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.


77. Suppose that Krusty’s note pays interest at 7%. If on April 20, 2009, Krusty needed cash and sold his note to Mel, who requires 9% on his investment, how much did Mel pay Krusty for his note?

78. On March 1, 2010, Troy (you might remember him from problem 54) bought at $15,000 180-day note. On April 20, 2010, Troy needed cash because his movie flopped, so he sold his note to Kent, who requires 13% on his investment. How much did Kent pay Troy for the note? Explain.

79. If Troy’s note pays 5% interest, and on April 20, 2010, Troy needed cash and sold his note to Superintendent Chalmers, who requires 13% on his investment, how much did Superintendent Chalmers pay for Troy’s note?

80. A corporate note is bought on 3/5/10 and matures to $5500 on 9/5/10. On 7/5/10, this note is sold to a third party requiring 6%.

(a) Find the amount the third person paid for the corporate note.
(b) Find the actual interest the original holder lost on the investment.
(c) Why would the original holder sell?

81. On 1/1/07, the city of North Haverbrook issues a tax-free $3000 municipal security that matures on 4/10/07 with interest at 6.5%. If the owner of a security sold it on 2/1/07 for $2850.75, what rate of return did the buyer get?

1.6 Equations of Value

Theorem 14 (The Golden Rule of Finance). Monies cannot be added or reconciled unless they are valued at the same point in time.

Therefore we need to move all money to one date to compare or calculate.
**Definition 15.** The *focal date* is the date to which monies are moved in an equations of value problem.

82. At 7%, what one amount of money in 2 years is equivalent to $500 in 3 months and $1000 in 6 months if we use today as the focal date? (Use time in months here.)

83. Patty owes $2000 due in 1 year and $3000 due in 5 years. What is her cash equivalent (i.e., payoff amount) of these two amounts at 3.5%? (Notice that this implies using today as the focal date.) (Use time in months here.)

84. Suppose that on 6/1/04, Homer buys a $500 sofa and agrees to make two equal payments on 12/1/04 and 6/1/05. If the store charges 10% for credit, find the amount of the 2 payments. Use a focal date of 6/1/05. (Use time in months here.)

85. Does the focal date matter? What if in the previous problem, we had used a focal date of 12/1/04 instead? (Use time in months here.)

86. The owner of the Leftorium (yes, that is Ned) has figured that his tax obligation for self-employment on 2/15/07 will be about $5800. He makes a $1500 payment on 8/15/06 and another $1600 payment on 11/15/06. What will he still owe on 2/15/07 if money is worth 3.5%? (Use time in months here.)

87. A debt of $7500 incurred on 5/10/07 must be paid back with two installments. The first is on 8/10/07 and the second, 50% larger, is on 11/10/07. If the interest rate is 8%, find the 2 payments. (Use time in months here, and 11/10/07 as the focal date.)

### 1.6.1 Equations of Value Homework Problems

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

88. At 12%, what one amount of money in 1 year is equivalent to $750 now and $850 in 6 months if
   (a) We use today as the focal date?
   (b) We use 6 months as the focal date?
   (c) We use one year as the focal date?

89. Selma owes $560 due in 1 year and $5280 due in 3 years. What is the cash equivalent of these two amounts at 7%? (Notice that “cash equivalent” tells you what to use as the focal date!)
90. Edna owes $1500 due in 1 year and $2189 due in 5 years. What is her cash equivalent of these two amounts at 2.5%?

91. Marge buys a $4000 car on 1/1/09 and agrees to make payments. If she pays $1000 down, another $1000 on 12/1/09, and another $1000 on 6/1/10, find the payoff on 12/1/10 if the loan is costing her 5%.

92. Homer pays $500 for Santa’s Little Helper (yes, I know - he didn’t pay anything for the dog....) on 6/1/09 and agrees to make three equal payments on 12/1/09, 6/1/10, and 12/1/10. If the breeder charges 8% for credit, find the amount of the three payments. Put the focal date at 12/1/10.

93. Repeat that problem, but with a focal date of 6/1/10.

94. Moe has figured out that his tax obligation for self-employment on 3/15/11 will be about $6500. If he pays $1500 on 9/1/10, and $2150 on 12/1/10, what will he still owe on 3/15/11 if money is worth 6%?

95. An obligation for $6130 is due on 11/13/11 and a second one for $5500 is due on 1/8/12. What single payment today will discharge these two obligations if money is worth 5%?

96. Krustylu Studios has two loans that come due in the current fiscal year. The first loan for $1.23 million was due August 1st and the second for $2.5 million is due November 1st. If the Studio missed the first deadline and is paying the second early, what is the payoff amount on September 12th if money is worth 5.25%? Use Banker’s Rule.
1.7 Partial Payments

**Definition 16.** *The U.S. Rule* calculates interest due on any day that a partial payment is made. The payment is then subtracted as cash on that day from the sum of principal and interest. This leaves a new balance (principal) that continues to accrue interest until the next partial payment or until the due date.

**Fact 1.** *Credit cards with no new charges work this way.*

97. Bart (acting on behalf of Santos L. Halper) borrows $1500 by taking a cash advance on his credit card. He makes the first three monthly payments of $300, but misses the fourth and fifth monthly payments. On the sixth month, he pays off the loan.

(a) If they are charging him 20%, find the payoff.

(b) Why would this never happen?

(c) What is the total amount that Santos pays?


(a) What is the outstanding balance on the loan on 8/10/09 at 5% using exact time and exact interest?

(b) What is the total that Agnes pays if it is paid off on 8/10/09?

(c) What is the interest that Agnes pays?

99. During her senior year at Springfield University, on January 20, 2010, Lisa borrowed $3300 from her father at 5%. She made the following payments:

- $400 on May 30
- $1580 on August 10
- $950 on October 5

Find the amount she still needs to pay him on December 1st to pay off her debt. Use exact time and exact interest.

100. A $15,000 car loan financed for 5 years at 9% has a monthly payment of $311.38. Use the US Rule to find out how much of each of the first two payments goes to interest and how much to principal.
1.7.1 HW problems about Partial Payments

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

For all problems, use US Rule.

101. Troy (you might remember him from homework problems 78 and 79) borrows $1500 and makes two monthly payments of $400. On the third month, he decides to pay off the loan. If Troy is being charged 19.5% on his credit card loan, find the payoff.

102. Edna borrows $1200 by taking a cash advance on her credit card. She makes the first three monthly payments of $300, but misses the 4th and 5th payments. On the 6th month, she decides to pay off the loan. If Edna is being charged 13.5%, find the payoff amount.

103. On January 1, 2010 Principal Skinner has a balance of $975 on his charge card at a rate of 21%. Suppose he pays the 15% minimum on the last day of each month. Track his balances and payments to the end of June using exact interest and exact time and the US Rule. What is his balance on June 30 after making his payment? Assume that no further purchases were made with his credit card during this time.

104. Selma purchased a used car on July 1 for $6000 and paid $1500 down. The dealer agreed to a 4% interest rate and two partial payments of $1500 on September 15 and December 5 with the stipulation that the balance be due on February 12. Find the final payment on February 12.

105. An $80,000 home loan financed for 30 years at 6% has a monthly payment of $532.24. Use the US Rule to find how much of the first two payments goes to interest and how much goes to principal.

106. If Ned makes quarterly tax installments of $860 on June 15, 2009; $1250 on September 15, 2009; and $625 on January 15, 2010; what will he still owe on March 15, 2010 to meet this $3580 tax obligation from March 15, 2010? Assume that money is worth 6%.
Chapter 2

Compound Interest

2.1 Basic Definitions and Compound Interest

Recall that simple interest is charged (or earned) only on principal.

**Definition 17.** Compound interest pays on the principal and previously earned (or accrued) interest.

We need several pieces of information in order to proceed with compound interest.

**Definition 18.** The *conversion period* or *interest period* is how often interest is earned on the money.

**Definition 19.** The *annual rate* is divided evenly over the conversion period.

**Definition 20.** The *interest rate per conversion period* is the rate per interest period, and is calculated by taking the nominal rate (annual percentage rate, or a.p.r.) divided by the number of periods per year.

107. Compute the future value of $100 for 4 years at 10% compounded annually (abbreviated 10%(1)).

108. Compute the future value of $100 for 4 years at:

   (a) 10%(2) (i.e., semiannually).
   (b) 10%(4) (i.e., quarterly).
   (c) 10%(12) (i.e., monthly).
   (d) 10%(52) (i.e., weekly).
   (e) 10%(360) (i.e., daily).

Before completing the last 3 exercises above, I hope that you figured out the following formula:
**Formula 5.** The future value for a compound interest problem is

\[ S = P(1 + i)^n \]

where \( S \) is the future value of the account, \( P \) is the principal, \( i \) is the interest rate per period (notice that this is different from \( r \) in the simple interest calculation!), and \( n \) is the number of times the money is compounded.

109. Homer owes $1000 to Springfield National Bank, who charges 6% compounded semiannually for 2.5 years. Find:

   (a) the value of \( P \), \( i \), and \( n \) with reference to Formula 5.
   
   (b) the amount that Homer owes in 2.5 years and find the interest he paid.

110. On February 2, 2010, Bob buys a $10,000 CD that matures on August 2, 2012 at 9%(2). Find the worth of the CD at maturity.

111. Suppose your savings account is paying 4%(12). On February 1, 2006 you have $382.50 in your account. How much remains in your savings account on August 1, 2011 when you withdraw $500?

### 2.1.1 HW problems for Compound Interest

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

112. Compute the future value of $750 for 3 years at 8%(1):

   (a) Using simple interest 3 times.
   
   (b) Using the compound interest formula.

113. Helen invests $2000 (on behalf of the church, of course) for 2 years at 8%. Find the amount if:

   (a) Money is compounded quarterly.
   
   (b) Money is compounded monthly.
   
   (c) Money is compounded daily (use 360 days per year).

114. Bob owes $60,000 to his lawyer for the latest murder trial. His attorney’s office charges 8% semi-annually. Find the amount Bob owes in 2.5 years and the compound interest he will pay.
115. On February 2, 2009, Monty buys a $100,000 CD that matures on August 2, 2015 at 9%(4). At maturity, Monty withdraws the money for a down payment on a Rolls Royce. How much does Monty have toward the car?

116. When Bart was born, Abe bought him a $250 savings bond that paid 5%. When Bart started ITT Tech at age 18, he cashed in the savings bond. How much did Bart get?

117. Repeat the above problem, but change the interest rate to 5%(4). How much more did Bart get?

118. Suppose your savings account is paying 4%(12). On February 15, 2006, you have $382.50 in your account. What remains in the account on December 1, 2011 when you withdraw $500? (Use simple interest for part of a period.)

119. On 9/1/07 you borrow $500 at 6%(2) and on 3/1/09 you pay $400. At that time, what is your outstanding balance?

### 2.2 Present Value and Discounting

120. Find the present value of $1500 due in 3 years at 9% converted quarterly. (i.e., how much do I need to deposit into an account now to have $1500 in 3 years to pay off that debt?)

121. Find the value on June 30, 2008 of a note whose maturity value is $3500 with maturity date September 30, 2011 at 6% converted monthly.

**Formula 6.** Notice that we don’t really need a new formula, and can continue to use

\[ S = P(1 + i)^n. \]

However, we can also solve this equation for \( P \) to get a present value formula, giving

\[ P = \frac{S}{(1 + i)^n} = S(1 + i)^{-n}. \]

122. Fat Tony has two notes due. The first is a $3500 note due on 1/1/12. The second is a $2000 note due on 11/1/10. Find the equivalent worth of these two debts on 7/1/10 discounting the $3500 note at 8%(12) and discounting the $2000 note at 7%(12).

123. Helen sees an investment that she thinks will be worth $25,000 in 10 years. What should she pay for this investment if she desires 9%(4) on her money?
124. Ned holds a note worth $7000 in 3 years and 2 months. What should a buyer desiring 8\%(4) (use simple interest for any remaining time) on his money pay for this note today?

125. Lisa holds a note worth $10,000 in 4 years and one month. What should a buyer desiring 10\%(4) on his money pay for this note? (Again, use simple interest for any remaining time.)

126. What would the buyer have to pay for Lisa’s $10,000 note if it was all simple interest?

2.2.1 HW problems for PV and Discounting

The following are problems to be completed as homework for extra practice with the ideas presented above. You should feel free to talk to other students or your teacher about these problems if you have any questions or trouble completing them.

127. Find the present value of $3200 due in 2 years at 8% converted quarterly.

128. Find the present value of $50,000 due in 4 years at 4% converted monthly.

129. Find the present value of $3200 due in 2.5 years at 3.5% converted semiannually.

130. Fat Tony owes a note of $10,000 in 3 years. What should a buyer desiring 8\%(4) on her money pay for this note?

131. Patty sees an investment that she thinks will be worth $5000 in 4 years. What should Patty pay for this investment if she desires 8\%(12) on her money?

132. Monty holds a note worth $4000 in 2 years and 3 months. What should a buyer desiring 6\%(12) on his money pay for this note?

2.3 Nominal Rates and Effective Interest

133. (a) Find the future value of $500 at 10\%(4) after one year.

(b) Find the actual interest earned.

(c) What simple interest rate would give us the same interest?

\textbf{Definition 21.} The \textit{annual effective rate} (a.k.a., annual percentage yield) is the simple interest rate that would yield the same interest as the nominal compound interest rate actually gives.
I find the following formula completely unhelpful, but include it in case you want to memorize something else.

**Formula 7.** The effective rate is

\[ i = \left(1 + \frac{i(m)}{m}\right)^m - 1 \]

where \( i(m) \) is the nominal rate, \( m \) is the number of conversion periods per year, and \( i \) is the effective rate.

134. Find the annual effective rate of 9\%(12).

135. Find the annual effective rate of 8\%(12).

136. Which has a higher annual effective rate: 12\%(12) or 12.3\%(2)?

137. Which has a higher annual effective rate: 8\%(4) or 8.1\%(2)?

138. How much additional interest is earned on $125,000 for 3 years if money is worth 6\%(12) instead of 6\%(1)?

139. Marge can get a $50,000 loan from one of two banks. The first charges 7\%(12) and the second charges 7.05\%(2). If she thinks she wants to pay off the loan in 5 years, which is better?

### 2.3.1 HW problems for Effective Interest

140. Find the annual effective rate to two decimal places of 8\% if the money is compounded:
   (a) semiannually
   (b) quarterly
   (c) monthly
   (d) daily

141. Find the annual effective rate to two decimal places of 12\% if money is compounded:
   (a) semiannually
   (b) quarterly
   (c) monthly
   (d) daily

142. Which is higher: 7.2\%(1) or 7\%(4)?

143. Which is higher: 7.2\%(2) or 7\%(12)?
144. How much additional interest would you get on $10,000 for 3 years if money is worth 6%(4) instead of 6%(2)?

145. How much additional interest would you get on $1000 for 4 years if money is worth 5%(12) instead of 5%(4)?

146. Homer can get a $4000 loan from two banks: Springfield Savings and Loan charges 9%(4) and the Shelbyville Savings and Loan charges 9.1%(2). If he wants to pay off his loan in 3 years, which bank is better and by how much?

147. Moe can get a $50,000 bar improvement loan from two banks: Shelbyville Savings and Loan charges 8%(12) and Ogdenville Savings and Loan charges 8.3%(2). If he wants to pay off his loan in 5 years, which bank is better and by how much?

2.4 Logarithms

**Definition 22.** We “undo” exponential functions by using logarithms. Therefore, the statement

\[ a^x = b \]

is the same as the statement

\[ \log_a b = x \]

meaning to what power do we have to raise the number \( a \) to get an output of \( b \)?

**Examples:**

- \( 2^x = 4 \) is the same as \( \log_2 4 = x \). The answer (i.e., the correct exponent) is \( x = 2 \).
- \( 3^x = 27 \) is the same as \( \log_3 27 = x \). The answer (i.e., the correct exponent) is \( x = 3 \) since \( 3^3 = 27 \).
- \( 10^x = 1000 \) is the same as \( \log_{10} 1000 = x \). The answer here is \( x = 3 \) since \( 10^3 = 1000 \). Notice that when we are talking about problems involving a base of 10 in the exponent, we use \( \log_{10} \) which is usually called the common logarithm and abbreviated \( \log \) with no subscript. Therefore if you see a logarithm with no subscript, it mean \( \log_{10} \).
- \( e \) is the number approximately equal to 2.718281828459045… and is frequently used in biology examples of continuous growth (and we will see it later in Section 2.8). If we have an exponential function involving \( e \), the logarithm that matches is \( \log_e = \ln \), the natural logarithm.
Fact 2. We can always use either log or ln to solve exponential equations, no matter what the base is in our exponential function. Why would we want to do this? Our calculators have two logarithm buttons - log and ln - and so if we want a decimal approximation we want to use one of these logarithms.

Fact 3. \( \log a^n = n \log a \)

148. Find simpler expressions for each of the following:
   (a) \( \log_4 16 \)
   (b) \( \log_{11} 121 \)
   (c) \( \log_e e^x \)
   (d) \( \log_9 81 \)

149. Solve for \( x \): \( 4 = 3^x \).

150. Solve for \( n \): \( 1000 = 500(1.06)^n \).

151. Solve for \( n \): \( 1500 = 500(1.03)^n \).

2.4.1 Homework problems about Logarithms

I will not include any homework problems solely about logarithms because our purpose is really to use them in our applications. Let's just move on to the next section and see how to use them to solve financial mathematics problems.

2.5 Finding the Time for an Investment to Grow

We use the logarithms given in the above section to find the number of compound periods needed for a certain future value. If you really need one (which I hope that you don't), we have the following:

Formula 8.

\[
    n = \frac{\log \left( \frac{S}{P} \right)}{\log(1 + i)}
\]

where \( S \) is the future value, \( P \) is the present value, and \( i \) is the interest rate per period. \( n \) is, of course, the number of investment periods.

152. How long does it take $500 to double at 6% (1)?

153. How long does it take $500 to triple at 12% (4)?
154. How long will it take $1000 to be worth $2385 at 8%(4) if
   (a) we answer to the nearest period?
   (b) No interest is given for a part of a period and the worth is at least $2385?
   (c) Use simple interest and Banker’s Rule for part of a period and
       answer to the nearest day.

155. How long will it take to increase our worth by 85% at 12%(1) if
   (a) we answer to the nearest period?
   (b) No interest is given for a part of a period and the worth is at least
       an 85% increase?
   (c) Use simple interest and Banker’s Rule for part of a period and
       answer to the nearest day.

2.5.1 HW problems for finding the growth time

156. How long will it take for $1500 to be worth $1750 at 8%(12) if:
   (a) you answer to the nearest n?
   (b) no interest is given for a part of a period and we need to have at
       least $1750?
   (c) you use simple interest and Banker’s Rule for a part of a period?

157. How long will it take to triple our worth at 10%(12) if:
   (a) you answer to the nearest n?
   (b) no interest is given for a part of a period and we need to have at
       least triple your money?
   (c) you use simple interest and Banker’s Rule for a part of a period?

158. Edna and Seymour need $20,000 for a down payment on a home. They
    have $15,000 in their savings account now. At 4%(12), how long will
    it take them to have their down payment? Answer all three ways.

159. Bob and Selma will need 10% down on their $250,000 home. They
    have $20,000 in their savings account (yes, Bob did rob a bank). At
    3.4%(12), how long will it be before they have their down payment?
    Assume that they need to have at least enough money for the down
    payment, but cannot put down less than 10%.

160. On June 1, 2002, Springfield University has $100,000 in their scholar-
    ship fund. When the fund reaches $250,000, the school plans to
    start distributing the scholarship. If the account earns 8% converted
    semianually, when will the school start distributing scholarships?
    Answer to the nearest day.
2.6 Equations of Value

Recall that for simple interest, the answers for equations of value were different depending upon the focal date. This is not true for compound interest (YAY!).

161. Milhouse owes $10,000 due in 3 years and $20,000 due in 5 years. At 9%(12) find the single payment that will cover both obligations if the payment is located now.

162. Find Martin’s payoff on 7/1/04 at 7% compounded semiannually if he owes $500 on 7/1/02 and on 1/1/03 he makes a $200 payment and on 7/1/03 he makes another $300 payment.

163. Ned deposits $7000 on his son’s 10th birthday into an education money market fund that has a yield rate of 7%(4). The fund is to be distributed in two equal amounts on the son’s 21st and 22nd birthdays. Find the amounts paid on those birthdays.

164. Helen’s trust fund of $250,000 is to be distributed to her grandchildren ages 5, 10, and 12. Her trust stipulates that each will get an equal amount in the year they turn 18. Find that amount if money is earning 4%(12).

Definition 23. The net present value is the “net” (i.e., difference) between income and expenses, all of which are moved to the present.

165. Find the net present value (NPV) at 20%(1) and 25%(1) for an investment that requires a $5000 outlay of cash now to receive a $4000 return in one year and a $2500 return in two years.

166. Is the above investment a good investment? Why or why not?

2.6.1 Homework problems for equations of value

167. Reverend Lovejoy owes $10,000 due in 3 years and $25,000 due in 5 years. At 8%(4), find the single payment that will cover both obligations if the payment is located:

(a) now
(b) in 4 years.
(c) in 8 years

168. Patty owes $500 on 7/1/09. On 11/1/09, she makes a $200 payment and on 4/1/10 she makes another $300 payment. Find her payoff on 7/1/10 at 7%(12).
169. Seymour charges $12,000 on his Visa card (to buy nice things for Edna) and makes three monthly payments of $2000. He desires to pay the credit card off the fourth month. Find the payoff if Visa charges 18\%(12).

170. Maude Flanders left the $150,000 proceeds from her life insurance to Rod and Todd (ages 10 and 8, respectively). Her will stipulates that each will get an equal amount in the year of their 21st birthday. Find the amount if money is earning 6\%(12).

171. A $3200, 8\%(2) note matures in 5 years. In 3 years it is sold to a third party who requires 5\%(4) on his money. Find the purchase price.

172. A $4500, 2-year note is sold in 1 year to a person requiring 7\%(4) on their investment. Find the proceeds.

### 2.7 Finding the Compound Rate

173. At what rate converted monthly is $200 worth $300 in 3 years? Express this both as a nominal rate and as a rate per month.

174. At what rate converted quarterly could Bart double his college (ha ha!) fund in just three years? Express both as a rate per period and as a nominal rate.

Again, if you really need another formula:

**Formula 9.**

\[
i = \left( \frac{S}{P} \right)^\frac{1}{n} - 1
\]

where i is the rate per period, S is the future value, P is the present value, and n is the number of compound periods.

175. At what rate converted yearly would a money market account increase by 50\% in just two years?

176. The budget for your marketing firm grew from $20 million in 1990 to $55 million in 2000. Find the percent growth and predict the budget in 2040.

177. Is this reasonable? Why or why not?

#### 2.7.1 HW problems for finding compound rate

178. At what rate converted monthly is $200 worth $500 in 4 years? Express the answer as a rate per month and as a nominal rate.
179. At what rate converted quarterly is $200 worth $500 in 4 years? Express the answer as rate per quarter and as a nominal rate.

180. At what rate converted annually is $200 worth $500 in 4 years? Express the answer as a rate per year and as a nominal rate.

181. What pattern can you see in the above three problems? Why does this make sense?

182. At what rate converted quarterly could Lisa’s bank account increase by 60% in just 2 years? Express the answer as a rate per quarter and as a nominal rate.

183. The budget for the First Church of Springfield has grown from $20,000 in 1990 to $35,000 in 2000. Find the percent growth and predict the budget in 2010. Is this reasonable?

2.8 Continuous Compounding

184. Find the amount of money in an account in one year if $10,000 is deposited today and the money earns 10%:

(a) compounded semiannually.
(b) compounded quarterly.
(c) compounded monthly.
(d) compounded weekly.
(e) compounded daily (use ordinary interest).
(f) compounded daily (use exact interest).
(g) compounded hourly.
(h) compounded every minute.

**Formula 10.** The future value for continuous compounding is

\[ S = P e^{\delta t} \]

*where* \( t \) *is measured in years and* \( \delta \) *is the rate of compounding. e is the number approximately equal to 2.718281828459045 . . . . . .*

(i) continuously? (i.e. \( 10\% (\infty) \))

185. Find the future value of $750 in 6 months at 8% compounded continuously. (i.e. \( 8\% (\infty) \))

186. If you need to have $400 in two years at 8% compounded continuously, how much should you invest now?
187. How long would it take for you to double your money at 10% compounded continuously (i.e., \(10\%(\infty)\))?

188. At what continuous compounding rate should you invest $500 to have $650 in two years?

189. Fat Tony invests in a $250,000 municipal bond that pays 8% compounded continuously and matures in 26 weeks.
   
   (a) What is the value of his investment if held to maturity?
   
   (b) If Fat Tony held the bond for 10 weeks and then sold it to Doris who wants a return of 10% compounded continuously, how much did Fat Tony get for the bond?
   
   (c) What rate of return (i.e., what continuously compounded interest rate) did Fat Tony make during the 10 weeks he owned the bond?

190. Three children ages 6, 8, and 10 are to receive equal shares of their grandfather’s estate of $165,000 when they reach 18. If the money is invested at 7.5% (\(\infty\)), what amount will they each receive at age 18?

191. What is the equivalent annual interest rate of 6% (\(\infty\))? 

2.8.1 HW for continuous compounding

192. Which earns more: 8%(\(\infty\)) or 8%(12) and why?

193. The Springfield Retirement Castle purchased a $250,000 piece of commercial paper (i.e., like a bond) paying 6.5% compounded continuously and maturing in 178 days. What is the value of the investment if held to maturity?

194. If the Springfield Retirement Castle held the commercial paper (in the problem above) for 65 days and then sold it to the Springfield Nuclear Power Plant, who wants to earn 7% compounded continuously, what did the Springfield Retirement Castle get for it?

195. Springfield Savings and Loan advertises CDs with 20 months maturity paying 5% compounded continuously. What is the maturity value of a $2500 CD?

196. Shelbyville Savings and Loan advertises CDs with 2 year maturity paying 6% compounded continuously. What is the maturity value of a $5000 CD?

197. Marge, Patty and Selma are to receive equal shares of their mother’s estate of $150,000 when they turn 50. If Marge is currently 34, and Patty and Selma are both 37, and the money is invested at 8% (\(\infty\)), find the amount that each will receive.

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198. What is the annual effective rate for $8\%(\infty)$?

199. What is the equivalent annual interest rate for $12\%(\infty)$?
Chapter 3

Annuities

3.1 Basic Definitions and Ordinary Annuities

**Definition 24.** An annuity is a sequence of payments (usually equal) dispersed or received at equal intervals of time.

Usually the payment intervals and compound interest intervals match. There are many common examples - think of mortgage, car loan, student loans, etc.

Each payment is some bit of the principal plus whatever interest accrued over the period.

**Definition 25.** The payment is sometimes called periodic rent.

**Definition 26.** Annuities certain begin and end at certain times.

**Definition 27.** Contingent annuities have beginning and ending dates that depend upon a specific event (life insurance, retirement funds, etc.)

**Definition 28.** Annuities in perpetuity have a specific starting time but an infinite number of payments (usually this consists of periodic interest paid out without touching the principal).

**Definition 29.** An ordinary annuity has payment at the end of each rent (i.e., compounding) period.

**Definition 30.** An annuity due has the payment at the beginning of each rent period.

**Definition 31.** A simple annuity has the interest compounded at the same frequency as the payments.

**Definition 32.** A general annuity has payments and conversion periods that don’t match.

We are going to focus our attention on ordinary annuities certain.
200. Find the future value of 4 semiannual deposits of $1000 each at 10%(2).
That wasn’t so bad to do without a formula, but we need one for more
complicated situations:

**Formula 11. The future value of an ordinary annuity certain is**

\[ S = R \left( \frac{(1 + i)^n - 1}{i} \right) \]

where \( i \) is the interest rate per period, \( n \) is the number of payments,
and \( R \) is the payment amount (a.k.a. rent).

201. (a) Find the future value of 30 monthly payments of $500 each at
6%(12).
(b) How much money is actually deposited?
(c) What is the interest earned?

202. Find the future value of 40 weekly payments of $45 each at 6%(52).
Be careful on the following problems.... There is a sneaky bit.....

203. On Maggie’s fifth birthday, Marge deposited $200 into a savings ac-
count earning 5%. Marge kept making these annual deposits until
Maggie’s 18th birthday.
(a) How many deposits were made?
(b) After that last deposit, how much money was in the account?

204. Do the same thing, but make $50 quarterly deposits into an account
compounded quarterly.

205. How about if you deposit $20 per month?

206. For each of the above, state how much was actually deposited into the
account and how much was earned in interest.

207. Lisa wants to have $10,000 in an account to pay for school. She de-
posits $100 per month at 6%(12) for 24 months, then $200 per month
for 24 months. How much does she need to deposit for the next 12
months to reach her goal?

### 3.1.1 HW problems for FV of ordinary annuities

208. Find the future value of 3 yearly deposits of $150 each at 12%(1):
(a) Using the compound interest formula three times.
(b) Using the future value of an annuity once.
209. Find the future value of 25 semiannual deposits of $250 each at 3.4\%(2).

210. Find the future value of 40 years worth of $20 monthly deposits at 3\%(12).

211. On Rod’s 4th birthday, Ned and Maude deposited $100 into a savings account paying 7\%. His parents continued such $100 deposits, making the last one on Rod’s 18th birthday. At that time, how much money was in the account?

212. On Todd’s 4th birthday, Ned and Maude deposited $25 into a savings account paying 6\%(4). His parents continued such quarterly $25 deposits and made the last one on Todd’s 18th birthday. At that time, how much money was in the account?

213. Who ends up with more money? Rod or Todd? Why?

214. Homer paid $1000 per year into his personal IRA from ages 30 to 45, inclusive.
   (a) If his fund is earning 8\%(1), how much money is in the IRA at age 45?
   (b) What is the total amount of money that Homer deposits into the IRA?
   (c) How much has he earned in interest on the money by the time he turns 45?
   (d) If he stops making deposits at age 45, but leaves the money in the account (still paying 8\%(1)) until he retires at age 65, how much money does he have at retirement?

3.2 Present Value of an Ordinary Annuity

Our motivating question in this section is: How much do we have to deposit today in a regular compound interest account to have as much in the account later as if we did an annuity?

215. Car Max advertises a vehicle for $2000 down and $400 per month for two years financed at 10.5\%(12). We want to know “what is the cash price of the vehicle?” Let’s work on this problem in pieces.
   (a) First find the future value of the annuity.
   (b) What would be the total paid for the car if you financed it?
   (c) What would the cash price be? In other words, what is the present value of that future value?
   (d) If you finance the car, how much do you pay in interest?
Problems can always be done this way, but it requires many steps. They are slightly easier using the following:

**Formula 12.**

\[
A = R \left( \frac{1 - (1 + i)^{-n}}{i} \right)
\]

where \(A\) is the present value, \(R\) is rent, \(i\) is the interest rate per period, and \(n\) is the number of payments.

216. (a) Edna is earning enough money as a teacher to afford to pay $1500 per month on a house. If the loan will have a term of 25 years at 6\%(12), for how large of a home loan will she qualify?

(b) How much will she pay in interest over the term of the loan?

(c) What if Edna has saved $20,000 towards a down payment?

(d) Edna found a $300,000 chalet in Ogdenville that she likes with the same down payment. If she cuts some other items out of her budget, allowing her to pay $2000 per month for her chalet on a 30 year mortgage at 6\%(12), can she afford her dream home?

217. Is it better to pay $20,000 in cash or to pay $1700 per month for a year at 8\%(12)? Why?

218. Suppose Moe has a 25-year “bar” loan with monthly payments of $1720 at 5.2\%(12).

(a) Find the cash value of his original loan.

(b) If Moe paid $200,000 down, what was the cash value of the bar when he bought it?

(c) If he wishes to pay off the loan after 16 years, find the payoff amount.

(d) Notice that he is more than half way through the loan and still owes more than half the money!

219. (a) Your student loan payment is $388.42 per month for 10 years. Find the present value of the loan at 2.722\%(12).

(b) If you wanted to pay off your student loan after 6 years, find the payoff amount.

### 3.2.1 HW problems for PV of ordinary annuities

220. Troy McClure (you might remember him from such problems as 78, 79, and 101) is earning enough to pay $500 per month on a house.
(a) If the loan will have a term of 20 years at 5%(12), how large of a home loan is Troy able to get?

(b) If Troy has saved $30,000 toward a down payment on the home, what will be the cash price of a home that he can afford?

(c) Troy has found a $250,000 home that he wants. Using the same down payment, but changing his payments to $800 per month at 6%(12), can he afford this home?

221. Is it better to pay $10,000 cash or $850.25 per month for a year at 8%(12)?

222. Is it better to pay $500 cash or to pay $21.22 per month for 2 years at 9%(12)?

223. Reverend Lovejoy has a home loan on which he pays $623 per month for 30 years at 5.5%(12).

   (a) What did he borrow for this home?
   (b) If he put $10,000 down when he purchased the home, what was the cash price of the home?

224. Suppose that Sideshow Bob has a 30-year mortgage with payments of $611.25 at 6.2%(12).

   (a) Find the cash value of Bob’s original loan.
   (b) If he paid $25,000 down, what was the cash price of the home?
   (c) If Bob wishes to pay off the loan after 25 years, what is the payoff amount? (HINT - how many payments does he have left at this point?)

225. Suppose Krusty bought a new home and has monthly payments of $811.25 at 6%(12) for 20 years.

   (a) What was the cash value of his loan?
   (b) If he paid $40,000 down on the loan, what was the cash price of his home?
   (c) If he wants to pay off the home after 12 years, how much does he owe?

226. Is it better to make monthly payments of $295.50 for 5 years or payments of $250.94 per month for 6 years to payoff a car loan of $16,637.96? Why?
3.3 Rent for an Ordinary Annuity

Here we need to solve for $R$. The hard part is knowing whether to use the present value or future value formula. Remember that the two formulas are:

\[ S = R \left( \frac{(1 + i)^n - 1}{i} \right) \]

and

\[ A = R \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

227. Within 3 years, Ned wants to save $20,000 for a down payment on some property. What would Ned’s monthly deposit be if his savings account is paying 4% (12)?

228. (a) Sideshow Bob borrows $45,000 at 12% (2) for a nice boat and wants to repay it within 3 years by making semiannual payments. Find the semiannual payment.

(b) If Bob can find someone to lend him the money at 6% (2), how much would he save per payment and what would his total savings be?

229. Chief Wiggum is buying a new police car costing $5200. After a $2000 down payment, he can finance the rest over 2 years with monthly payments at a rate of 3.4% (12). Find his monthly payment.

230. Milhouse is considering a new mountain bike that sells for $3500 but decides to save his money to buy it. He hopes to have all of the money saved in 3 years. His savings account pays 3% (12). Find his monthly deposit.

3.3.1 HW problems for finding rent

231. (a) Find the monthly payment for a car loan of $16,637.96 at 2.53% (12) for 5 years.
(b) What is the total amount repaid?
(c) How much is paid in interest?

232. (a) Find the monthly payment for a car loan of $16,637.96 at 2.75% (12) for 6 years.
(b) What is the total amount repaid?
Annuities

(c) How much is paid in interest?

233. Within 5 years, Principal Skinner wants to move out of his mother’s house, and wishes to save $10,000 for a down payment on his own home. What would his monthly deposit be if his savings account is paying 4%(12)?

234. (a) Lisa borrows $45,000 at 6%(12) for her college education and wants to repay it within 10 years by making monthly payments. Find her monthly payment.

(b) Would it be better or worse for Lisa to make quarterly payments if the loan is charging 5.75%(4) and she still wants to repay the loan in 10 years? Explain.

235. Ned found some left-handed ovens that he wants to install in his kitchen. The oven costs $750. After a $200 down payment, it can be financed over 1 year with monthly payments at 2%(12). Find his monthly payments.

236. (a) Barney wants to buy a new motorcycle that sells for $25,250 and decides to save the money rather than going into debt to buy it right now. He wants to save the money within 2 years. If his savings account pays 3%(12), how much must he deposit in the account each month to get the money within that time?

(b) What if Barney decides he can wait 5 years for the motorcycle? Then what would his monthly deposits have to be?

(c) Two years into Barney’s 5-year plan, he gets impatient and withdraws all of the money that he has to buy a motorcycle. How much money can he spend?

3.4 Extra Practice for Finding Rent

237. (a) Max borrows $42,500 at 12% (2) for a nice boat and wants to repay it within three years by making semiannual payments. Find the semiannual payment.

(b) If Max can find someone to lend him the money at 5.5% (2), how much would he save per payment and what would his total savings be?

238. Within three years Jolene wishes to save $30,000 for a down payment on some property. What would Jolene’s monthly deposit be if her savings account is paying 4.5%(12)?
239. (a) James is buying a new HVAC system costing $6500. After a $1500 down payment, he can finance the rest over two years with monthly payments at a rate of only 3.5% (12). Find his monthly payment.

(b) If James wants to pay off the HVAC system after 6 months, what is his payoff amount?

240. (a) Rosa is considering a new motorcycle that sells for $15,000 but decides to save her money to buy it. She hopes to have all the money saved within three years. Her savings account pays 3% (12). Find her monthly deposit.

(b) If Rosa decided to finance the bike instead, and pay off the loan in three years at 3%(12), what would her monthly payment be?

241. A $10,000 loan for 36 months at 6% (12) is arranged for the customer to make payments of $R$ for the first year, $2R$ the second year and $3R$ the third year. Find the payments for each of the three years.

242. How much needs to be in an education fund on August 1 in order to provide $800 per month for 9 months for each of the following 4 years of college with payments made at the beginning of each month starting in September? Assume the fund earns 5.5% (12).

### 3.5 Term of an Ordinary Annuity

**Definition 33.** Remember that the term of an ordinary annuity is the number of payments for that annuity.

Like in the previous section, the hard part here is deciding whether to use future value or present value formulas.

243. How many half-years will it take for a deposit of $300 per half-year to accumulate to $5000 at 6%(2) if

   (a) we round to the nearest half year?

   (b) we need at least $5000?

   (c) we compute the number of full $300 deposits and find the final deposit to reach $5000.

244. How many months of $500 payments will be needed to retire a VISA card debt of $5050 at 12.3% (12) if we

   (a) answer to the nearest month?

   (b) need to completely pay it off?
(c) find the number of full $500 payments and the size of the last payment.

245. How long will it take Moe’s Bar, which is depositing $250 per month into a sinking fund earning 5.1% (12) to accumulate $10,000 to redecorate the bar?

246. Act as a financial planner for Moe and set up a monthly deposit plan so that he will have the necessary money in 15 months.

### 3.5.1 Homework problems for finding the term

247. How many quarters will it take for a deposit of $125 per quarter to accumulate to $4500 at 6%(4) if we:

(a) round to the nearest quarter?
(b) get no interest for a part of a quarter and need to make at least $4500?
(c) compute the number of full $125 deposits and find the final deposit?

248. How many months of $250 payments will be needed to pay off a debt of $5600 at 9.9%(12)? Answer all three ways.

249. (a) Marge deposits $25 per month into an account paying 5.5%(12) for Lisa’s college education. If she wants to have at least $10,000 in the account, how many deposits must she make?
(b) If Marge wants to have $50,000 in the account (to almost pay for one year at Springfield University), how many deposits must she make?
(c) If Marge needs to have the $50,000 in 10 years, how much should she be depositing into that account every month?

250. The settlement for Homer’s worker’s compensation case (see “King-size Homer”) is $125,000. He invests that money (yeah, right!) at 8.5%(12).

(a) How many monthly $2000 monthly payments will he receive from this settlement?
(b) What will the smaller last payment be?

251. Luann van Houten assumes the mortgage from Kirk when they get divorced. The outstanding balance on the house is $68,850, and she agrees to pay $650 per month at 5.5%(12).

(a) For how many months will she have to make payments?
(b) What will the smaller last payment be?

252. Comic Book Guy has $50,000 for a down payment (he sold some of his classic comics that he was keeping in his mother’s attic) and can afford $2050 per month toward the purchase of the Android’s Dungeon comic book shop. If the building cost $250,000, how long will it take him to pay it off at 12% (12)?

## 3.6 Interest Rate of an Ordinary Annuity

253. At what interest rate will $700 per half year satisfy a loan of $2000 in 2 years?

254. If I buy a car for $16,500 and make 60 monthly payments of $312, what interest rate am I being charged?

You may have noticed - these are guess and check problems. We will talk about how to have a computer do these for you, but in the mean time you should develop a system that helps you give better and better guesses.

255. If you can accumulate $10,000 over 2 years by investing $1000 per quarter, what rate of return are you getting?

### 3.6.1 HW problems for finding interest rate

256. At what rate of interest will $100 per month be worth $2500 in 2 years?

257. At what rate of interest will $250 per month be worth $6250 in 2 years?

258. At what rate of interest will $100 per month pay off a debt of $2138 in 22 months?

259. If Agnes pays off a $10,000 car by making two $5500 semiannual payments, what interest rate is she being charged?

260. If Abu can save $12,000 over 19 months by making deposits of $550 each month, what interest rate is he earning?

261. Ned has $15,000 in a savings account that pays 5.4% (4). He plans on opening a newer and bigger Leftorium in 3 years. To do this, he deposits $1500 per quarter into another account. If he needs to have a total of $35,000, what does the new account need to pay?
3.7 Debt Retirement Methods

Definition 34. To amortize a debt is to pay a sequence of equal size payments, each of which is composed of the interest due plus a portion of the principal. Payments are structured to match the conversion frequency of the compound interest rate.

Definition 35. The cash price is the price you pay, including taxes.

Definition 36. The down payment is the actual money that you pay on the home (or car) before borrowing any money.

Definition 37. The closing costs are the charges associated with the borrowing process. These are fees to process paperwork and pay staff, legal fees, filing fees, title search and title insurance.

Fact 4. That payment for a loan is based on the compounded interest rate and the term of the loan. The principal is thought of as the present value of an ordinary annuity.

Definition 38. An amortization schedule is a table giving the following information about each payment:

- the number of the payment or date of the payment
- the dollar value of the payment
- the interest due at that payment
- the amount of that payment that will go to the principal
- the balance of the loan

262. Marge buys a $30,000 car with a $5000 trade-in and an additional $2000 down. The balance is financed over 3 years at 3.8%(12).

(a) How much is financed?
(b) Find the monthly payment.
(c) What is the total amount repaid on the loan?
(d) Find the total paid in interest.
(e) Fill in the first 5 lines of the amortization table:

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount Owed</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Balance</th>
</tr>
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</tbody>
</table>
(f) How do you find the 12th line without doing all of it? (Hint - 11 payments have been made and so 25 payments remain.....). Find the 12th line of the amortization table.

3.7.1 HW Problems for Debt Retirement Methods

263. A car costs $20,500. You have a $1000 trade-in and can pay $3000 down.

(a) What is the amount that needs to be financed?
(b) You will pay the car off in 60 equal monthly payments at 2.65%(12). What will your payment be?
(c) What is the total amount that you will repay?
(d) How much will you pay in interest?
(e) What would the payment be if you financed at 2.75%(12) for 72 months?
(f) What is the total amount that you will repay?
(g) How much will you pay in interest?
(h) If you took the 2.75%(12) loan, but decided to pay off the car after 60 months, how much will you still owe?

264. (a) Find the loan payment for a $150,000 home requiring 10% down, financed at 5.6%(12) for 20 years.

(b) How much would you still owe if you wanted to payoff the loan after 15 years?
(c) How much of the first payment goes to interest?
(d) How much of the first payment goes to principal?
(e) What is the total amount that you will repay?
(f) How much will you pay in interest?

265. You owe $35,000 in student loans when you graduate from school.

(a) If the loans are borrowed at 4.5%(12), what will your monthly payment be on the 10 year repayment plan?
(b) How much of the first payment goes to interest?
(c) How much of the first payment goes to principal?
(d) How much do you owe after the first payment?
(e) How much of the second payment goes to interest?
(f) How much of the second payment goes to principal?
(g) What is the total amount that you will repay?
(h) What is the total amount of interest you will pay?

(i) If you decide to pay off your student loans after 5 years, how much will you still owe?

(j) If you decide to pay off your student loans after 7 years, how much will you still owe?

266. What if you owe $50,000 at 3.5% (12)? Answer all of the same questions for a 10 year repayment plan.

267. What if you want to take 20 years to repay the $50,000 student loan at 3.5% (12)? Answer the same questions.

268. Sideshow Bob borrows $45,000 at 12% (2) for a nice boat and wants to repay it within three years by making semiannual payments.

(a) Find the semiannual payment.

(b) Find the total amount that Bob will repay.

(c) Find the total amount of interest that Bob will pay.

(d) Fill in the complete amortization table for Bob’s loan:

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<tr>
<th>Month</th>
<th>Amount Owed</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
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269. The Flanders, who just bought a new house, are paying $500 per month on a 20 year loan and the Springfield Mortgage Company is charging 7.3% (12).

(a) Find the cash value of the loan.

(b) What is the total amount of interest that they will pay?

(c) If they want to pay off the loan after 12 years, find the payoff amount.
(d) Fill in the first 6 lines of the amortization table. See above for an example.

(e) Find the 14th line of the amortization table.

And that brings us to the end of the course. Study hard for your final!
Appendix A

Tables and Formula Sheets

The following sections provide other useful information for the course. Most importantly this includes the serial table (in Section A.1 and a formula sheet in Section A.2 (you will see these repeatedly during the course). Also included in the last section are some extra practice problems for annuities and amortization. Feel free to use these (non-Simpson’s themed) problems if you need some more practice with these concepts.
### A.1 Serial Table

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</table>
A.2 Formula Sheet

\[ S = P(1 + rt) \]

\[ P = \frac{S}{1 + rt} \]

\[ S = P(1 + i)^n \]

\[ P = \frac{S}{(1 + i)^n} = S(1 + i)^{-n} \]

\[ i = \left(1 + \frac{i(m)}{m}\right)^m - 1 \]

\[ i = \left(\frac{S}{P}\right)^\frac{1}{n} - 1 \]

\[ n = \frac{\ln \left(\frac{S}{P}\right)}{\ln (1 + i)} \]

\[ S = Pe^{\delta t} \]

\[ P = Se^{-\delta t} \]

\[ i = e^{\delta} - 1 \]

\[ S = R \left(\frac{(1 + i)^n - 1}{i}\right) \]

\[ A = R \left(\frac{1 - (1 + i)^{-n}}{i}\right) \]
A.3 Extra practice problems for annuities

Answer each of the following questions.

1. On Joe’s 5th birthday, his mother deposited $200 into a savings account earning 5%. She kept making these annual deposits until his 18th birthday.

   (a) After that last deposit, how much money was in the account?
   (b) What was the total amount deposited?
   (c) How much interest was earned?
   (d) Do the same, but make quarterly deposits into an account paying 5% compounded quarterly.
   (e) What was the total amount deposited?
   (f) How much interest was earned?
   (g) Do the same, but deposit $50 per month into an account paying 5% compounded quarterly.
   (h) What was the total amount deposited?
   (i) How much interest was earned?

2. A woman deposits $200 at the end of each 3 months in a bank that pays 5% converted quarterly. How much will she have to her credit at the end of 10 years?

3. In 1974 a woman put $500 in common stocks. She continues to make the same deposit every year. If the stocks increase in market value at about 4% per year, what will be the value of the woman’s portfolio just after she makes her purchase in 1991?

4. You want to have $10,000 in an account to pay for school. You deposit $100 per month at 6% (12) for 24 months, then $200 per month for 24 months. How much do you need to deposit for the next 12 months to reach your goal?

5. You want to have $1,000 in an account to pay for a car. You deposit $25 per month at 6% (12) for 24 months, then $50 per month for 24 months. How much do you need to deposit for the next 12 months to reach your goal?

6. The settlement on a workman’s compensation policy of $1,000,000 is invested at 6.5% (1). How many $200,000 yearly payments could the beneficiary expect to receive from this settlement? What will the smaller last payment be?
7. A family assumes an $80,000 mortgage at 7%(12) by agreeing to pay $1000 per month. How many $1000 payments will they need to make and what will their smaller last payment be?

8. An executive has $100,000 for a down payment and can afford $2000-per-month house payments. How long will it take her to pay off a $250,000 home at 8% converted monthly?

9. How long will it take Kathy to accumulate $5000 by depositing $500 every 6 months at 5% compounded semiannually?

10. If a college student has $56,000 in her education fund on September 1 of her freshman year, find her monthly income from this fund under the following set of conditions: The fund is earning 6% (12), she only takes income from the fund from September 1 to May 1 of each school year, and she attends college 4 years prior to graduating.

11. If you can accumulate $10,000 over 2 years by investing $1000 per quarter, what rate of return are you getting?

12. A $30,000 car can be bought with 20% down, a $5000 trade-in, and a loan requiring $600 per month for 3 years. Find the APR of the loan.

13. A young couple saved the 15% down payment on their new $150,000 home by making $300 monthly deposits over the last 5 years. Find the rate their investment account is paying.

14. Within three years Jolene wishes to save $20,000 for a down payment on some property. What would Jolene’s monthly deposit be if her savings account is paying 4%(12)?

15. Max borrows $45,000 at 12% (2) for a nice boat and wants to repay it within three years by making semiannual payments. Find the semiannual payment.

16. If Max can find someone to lend him the money at 6% (2), how much would he save per payment and what would his total savings be?

17. James is buying a new HVAC system costing $5200. After a $2000 down payment, he can finance the rest over two years with monthly payments at a rate of only 3.4% (12). Find his monthly payment.

18. Rosa is considering a new mountain bike that sells for $3500 but decides to save her money to buy it. She hopes to have all the money saved within three years. Her savings account pays 3% (12). Find her monthly deposit.

19. A $10,000 loan for 36 months at 6% (12) is arranged for the customer to make payments of $R for the first year, $2R the second year and $3R the third year. Find the payments for each of the three years.
20. How much needs to be in an education fund on August 1 in order to provide $800 per month for 9 months for each of the following 4 years of college with payments made at the beginning of each month starting in September? Assume the fund earns 5.5% (12).
Appendix B

Practice Exams

The following are actual exams given during the Fall 2010 semester in this course. You will notice that you are scheduled to have two exams plus a final in this semester’s course, but there are only two practice exams included here. The third has been lost on my computer somewhere, so you will have to make due without a practice second exam (unless it miraculously resurrects itself). Use these as a guide for studying - maybe even study for a while first, then sit down and try to take the exam below in 50 minutes without the use of a book or notes and see how you do. Good luck!

B.1 Exam 1

Please write your name only on this page of the exam and on no other page of the exam.

Show all of your work to receive credit, and none of anyone else’s. The final answer is worth one point. The rest of the credit comes from your work.

If your cell rings, buzzes or anything else, you will receive a zero. Same zero if I find you doing anything resembling cheating.

1. (8 points) A woman borrows $30,000 to buy a home. The interest rate is 12% and the monthly payment is $308.59. How much of the first payment goes to interest and how much to principal? (Notice that during the first month this is a simple interest problem.)

2. (13 points) On May 4, 2007, a person borrows $1850 and promises to repay the debt in 120 days with interest at 12%. If the loan is not paid on time, the contract requires the borrower to pay 10% on the unpaid amount for the time after the due date. Determine how much this person must pay to settle the debt on December 15, 2007.
3. (10 points) If you can earn 7.5% on money, is it better to pay $1990 cash for an item or to pay $2090 in a year? Explain your answer.

4. (10 points) A person owes $350 due in 3 months and $525 due in 6 months. If money is worth 16%, what single payment in 6 months will settle both obligations? Put the focal date at 6 months.

5. (12 points) A couple wins $100,000 in a lottery and wishes to invest it at 10% simple interest to produce equal amounts of money for each of their two children, now ages 11 and 16, when the children reach age 21. What is the amount each child will get? Put the focal date at 10 years from now.

6. (15 points) A company wants to invest $500,000 from July 15 until September 28. The company has a choice of four banks in which to invest the money. All four will pay 12% simple interest. Bank A will use ordinary interest and approximate time, Bank B will use exact interest and approximate time, Bank C will use exact interest and exact time, and Bank D will compute using Bankers’ Rule. Find the interest each bank will pay on the investment and indicate which bank the company should choose.

7. (10 points) If the terms of a $5000 purchase are 3/30, n/60, how much would the buyer save on the 60th day if money was borrowed from a bank on the 30th day for 30 days at 12% to take advantage of the cash discount?

8. (10 points) Joe borrowed $2000 from Joseph at 15% to be repaid in 9 months. Three months after the loan, Joseph sold the note to Josephine who wants to get 18% on her money. Find the selling price of the note.

9. (12 points) On 2/5/08 Pierce has $10,000 in a savings account that is earning 5%. The next date that interest is added to the account is 6/1/08. If Pierce withdraws his money early, he will lose all of the interest on the amount withdrawn. On 4/8/08 Pierce needs $8000 for medical bills. If he can get a loan of 10%, should Pierce withdraw the money from his savings account or should he get the loan?

B.2 Exam 1 - Another Example

1. (12 points) Randall buys a $500 chocolate lab puppy from a championship line on 6/10/04 and agrees with the breeder to make two equal payments on 12/1/04 and 6/1/05. If the breeder charges 10% for credit, find the amount of the two payments. Use the focal date of 6/1/05.
2. (10 points) You deposit $550 into an account paying 8%\(^\text{r}\). Find the maturity value of the investment in 3 years and 5 months, using simple interest for a part of a period.

3. (10 points) On September 25, 2004, a 180-day T-bill is bought with a bid of 98.5%. Find the ROR (rate of return) on this investment.

4. A $4000 corporate note paying 4.5% interest is bought on 3/5/04 and matures on 9/5/04. On 7/5/04 this note is sold to a third party requiring 7%.
   (a) (10 points) Find the amount the third person paid for the corporate bond.
   (b) (3 points) Find the actual interest the original holder made from the investment.
   (c) (6 points) What was the original investor’s ROR?

5. (12 points) Reliance Hardware owes their paint supplier $12,800 on April 1. The supplier will agree to three payments of $4000 on June 1, $3500 on August 1, and $2500 on October 1, provided that the balance is paid on December 1, and Reliance Hardware pays 8% interest. Find the final payment.

6. (10 points) The Classic Construction Company (CCC) receives an invoice from Upstate Builder’s Supply for $59,865 with terms 2/10, n/45. If CCC pays the invoice on the 8th day, what rate of return will they make?

7. (8 points) If money costs 8% is it better to pay $3000 in cash or to wait 6 months and pay $3200? Explain your answer.

8. (8 points) If an investment paying 7.25\%(52) is expected to be worth $7500 in 3 years, how much should you pay for it today?

9. A $750 savings bond is purchased on your date of birth.
   (a) (5 points) How much is it worth on your 21st birthday if it pays 6%?
   (b) (6 points) How much more would it be worth if it paid 6\%(2)?
B.3 Exam 1 - A Third Example

1. (10 points) Suppose your savings account is paying 4%(12). On 2/1/02 you have $375 in your account. How much remains in your account on 2/1/08 after you withdraw $400?

2. (10 points) A 182-day T-bill is bought with a bid of 90.9%. Find the ROR (rate of return).

3. (12 points) On 7/1/04, Jim bought a $4000, 90-day note. On 8/20/04, Jim needed cash and sold his note to Pete, who requires 10% on his investment.
   (a) How much did Pete pay Jim for the note?
   (b) If Jim paid $3800 for the note, what is his ROR?

4. (10 points) Laura borrows $5000 on 3/10/05 and pays back $4000 on 6/10/05. What is the outstanding balance on the loan on 8/10/05 at 5%?

5. (10 points) If a general contractor receives a building materials invoice for $94,850 with terms 4/10, n/60, what rate of interest does he earn by paying on the 8th day?

6. (10 points) If money earns 6.25% simple interest, is it better to buy computer updates for $23,000 cash or for $25,000 in 15 months. Explain your answer.

7. (12 points) If an investment paying 8%(4) is expected to be worth $10,000 in 3 years and 2 months, how much should you pay for the investment today? (Use simple interest for a part of a period.)

8. A $500 savings bond is purchased on your date of birth.
   (a) (5 points) How much is it worth on your 20th birthday if it pays 6%?
   (b) (7 points) How much more would it earn if it paid 6%(1)?

9. (14 points) Homer has a debt of $2000, charging 8%, due on 6/10/05 and a debt of $3250, charging 7%, due on 8/15/06. What one amount on 12/1/05 will discharge both debts? Use 12/1/05 as the focal date.
B.4 Exam 2 - A First Example

1. (6 points) What amount of money will be required to repay a loan of $6000 on December 31, 1994 if the loan is made on December 31, 1988, at a rate of 10% compounded semiannually?

2. (10 points) Which gives the better annual return on an investment: 6\(\frac{1}{8}\)% converted annually or 6% converted quarterly? Explain your answer.

3. (8 points) A principle of $900 earns 6% converted quarterly for 3 years, and then 7% converted semiannually for 2 more years. Find the final amount.

4. (8 points) Find the present value of $7500 due in 4 years if money is worth 14% compounded monthly.

5. (12 points) A note dated June 1, 1995 calls for the payment of $85,000 in 6 years. On March 1, 1998, the note is sold at a price that will yield the buyer 12% compounded monthly. How much is paid for the note?

6. (10 points) If $500 amounts to $700 in 5 years, with interest compounded quarterly, what is the nominal rate of interest on the investment?

7. (12 points) Jamil can get a $50,000 home improvement loan from two banks. The first charges 9%(4) and the second charges 9.1%(2). If he wants to pay off his loan in 2 years, which bank has the lower effective rate? Explain.

8. (12 points) In what time will $2000 amount to $2500 at 5% compounded monthly? Allow simple interest for a fraction of a period and get the answer to the nearest day.

9. (14 points) A man leaves an estate of $30,000 that is invested at 7% compounded continuously. At the time of his death, he has three children, aged 13, 15, and 19. Each child is to receive an equal amount from the estate upon reaching age 21. How much does each get?

10. (8 points) $5000 is invested for 18 months at a nominal annual rate of 8%. Find the amount of interest earned if interest is compounded continuously.

B.5 Exam 2 - A Second Sample

1. (12 points) What rate of interest gives the higher annual effective rate: 8% compounded quarterly or 8.2% compounded annually? Explain.
Practice Exams 54

2. (14 points) What is the maturity amount of $40,000 for 6 years and 3 months at 10% converted semiannually? Use simple interest for a part of a period.

3. (10 points) A savings and loan association in California advertises that it is paying 5.25% converted daily (using Banker’s Rule). An investor in Ohio has money in an account paying 5% converted quarterly. If the Ohio investor transfers $10,000 to the California association, how much additional interest will be earned in a year?

4. (8 points) A debt of $250,000 is due on September 15, 1995. What payment must be made on March 15, 1993 to repay the debt if the borrower is permitted to discount the debt at 12% compounded semiannually?

5. (12 points) A woman can buy a dining room set for $2800 cash or for payments of $1500 down and $1500 in 2 years. If she can earn 7.5%(1) on her money, which plan is better?

6. (10 points) If $7500 accumulates to $12,000 in 6 years, find the nominal rate if interest is compounded semiannually.

7. (12 points) An alumnus leaves $75,000 to a university with the provision that it is to be invested until it amounts to $100,000. From then on the interest is to be used each year for scholarships. If the school gets 9% compounded annually, to the nearest day, how long must the gift remain in the fund to first amount to $100,000?

8. (12 points) A building is sold for $35,000. The buyer pays $8000 cash and signs a note with maturity value $10,000 due in 1 years and another with maturity value $10,000 due in 2 years. If the seller charges interest on the debt of 10%(4), what should be the maturity value of a third note due in 3 years that will pay off the remainder of the debt?

9. (10 points) An obligation of $10,000 is due in 5 years. Find the present value of the money at 7% compounded continuously.

B.6 Final Exam

1. (5 points) A worker borrowed $150 from February 2, 2004 until April 2, 2004 and paid $9.00 interest. What was the annual rate? (Use Banker’s Rule.)

2. (5 points) Suppose that you borrow $100 at 12.5% simple interest but the lender refuses to take payment until the time when you must repay $200. How long will it take? (Use Banker’s Rule if needed.)
3. (8 points) If the terms of a $4000 purchase are 3/30, n/60, how much would the buyer save on the 60th day if money was borrowed from a bank on the 30th day for 30 days at 12% to take advantage of the cash discount? (Use Banker’s Rule.)

4. (5 points) In 1626 Peter Minuit bought Manhattan Island from the Indians for trinkets valued at 60 guilders, or $24. Had this money been put into a savings account paying 3% converted annually, how much would be in the account now?

5. (4 points) Find the effective rate corresponding to 12% compounded continuously for one year.

6. (10 points) A loan of $3000 is partially repaid with payments of $1000 after six months and $1500 after one year. If the rate is 12% converted semiannually, find the payment in 2 years that will discharge the debt.

7. (12 points) The parents of four children, a 13-year old, a 12-year old, and 10-year old twins, die suddenly. Their wills call for a trust fund of $200,000 to be held, with equal amounts of the money to be given to each child at age 18. If the fund earns 10% converted quarterly, find the amount each child should get.

8. (8 points) How long will it take an investment of $1000 to be worth at least $3000 at 6% converted monthly?

9. (6 points) A television set is bought for $50 cash and $18 a month for 12 months. What is the equivalent cash price if the interest rate is 24% converted monthly?

10. (8 points) Mr. Freeman can afford to set aside $2000 every 6 months from his salary. He plans to buy a resort cottage when he has accumulated $20,000. How long will it take him if he can invest the money semiannually at 5%(2)? Give the answer to the nearest payment.

11. (12 points) A family has $3000 in savings. If they get 6% compounded semiannually, and withdraw $500 at the end of each 6 months, how many withdrawals can they make? What will be the size of the concluding withdrawal if it is made 6 months after the last full withdrawal?

12. Maxine borrows $45,000 at 12%(12) for a nice boat and wants to repay it within 6 months by making monthly payments.

   (a) (5 points) Find the monthly payment.
   (b) (3 points) Find the total amount that she will repay.
   (c) (3 points) Find the total amount of interest that she will pay.
(d) (6 points) Fill in the complete amortization table for her loan:

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount Owed</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Balance After Payment</th>
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Appendix C

Answers

The following is a list of answers for the in-class and homework problems in this text. All answers given should be suspect - I have not intentionally misrepresented any answers, but typos (or calculator errors on my part) are possible. If your answer doesn’t match the one given below, you should ask a question about it during class or office hours, but do not assume that your answer is incorrect if you have checked your work.

Chapter 1

Section 1.1
1. $I = 54.67$
2. $I = 381.11$
3. $I = 168.75$ and $S = 5168.75$
4. $I = 1458.33$
5. $r = 0.1 = 10\%$
6. 16 years and 8 months

Section 1.1.1
7. $I = 17.33$
8. $I = 68.42$
9. $I = 1736.54$
10. $I = 212.50$
11. $I = 63.75$ and $S = 1063.75$
12. $I = 210$ and $S = 1210$
13. \( I = \$79.17 \) per month, so no.

14. \( r = 156\% \)

15. \( t = 12.5 \), so 13 months

16. 20 years

Section 1.2

17. See the serial table.

18. January - 31
   February - 28 (29 in a leap year)
   March - 31
   April - 30
   May - 31
   June - 30
   July - 31
   August - 31
   September - 30
   October - 31
   November - 30
   December - 31

19. 243 days. (244 in a leap year).

20. \( 288 - 45 + 1 = 244 \) days

21. The year is divisible by 4. Except if it is also divisible by 100.

22. Years also divisible by 100 are not leap years, unless they are also divisible by 400.

23. 365 (366 in a leap year)

24. 240 days, and this is always the same number of days

25. 241 days, and this is always the same numbers of days

26. 360 days

27. (a) \( I = \$102.08 \)
   (b) \( I = \$101.25 \)
   (c) Nope
   (d) The one that charges less interest on the loan, so the one using approximate time.

28. (a) \( I = \$50 \)
(b) \( I = \$50 \)
(c) \( I = \$50.69 \)
(d) Banker’s Rule since it makes a year to be a fraction greater than 1.

29. \( r = 7.81\% \)

**Section 1.2.1**

30. Yes since it is divisible by 4.
31. Yes, since it is divisible by 400.
32. Approximate time is 265 days.
   Exact time is 268 days.
33. Approximate time is 51 days.
   Exact time is 51 days.
34. (a) \( I = \$47.26 \)
   (b) \( I = \$46.23 \)
   (c) \( I = \$46.86 \)
   (d) \( I = \$47.92 \)
35. \( I = \$272.22 \) and \( S = \$14,272.22 \)
36. \( r = 2.25\% \)
37. \( r = 21.05\% \)
38. \( S = \$1226.03 \)

39. He should take the loan (he ends up ahead by $2.08).

**Section 1.3**

40. \( S = \$3420 \)
41. \$7806.66
42. \( r = 20.99\% \)
43. \( r = 20.24\% \)
44. \( r = 19.86\% \)

**Section 1.3.1**

45. \( S = \$1040 \)
46. \( S = \$2160 \)
47. \( S = \$588.60 \)
48. \( \$1026.78 \)
49. \( \$43,170.960 \)
50. \( S = \$1520.22 \)
51. \( r = 22.27\% \)
52. \( \$1701.54 \)
53. \( S = \$5858.45 \)

**Section 1.4**

54. \( P = \$18,980.48 \)
55. Payoff amount is \( \$3179.85 \)
56. Better to pay \( \$3200 \) in 6 months.
57. Better to pay \( \$2200 \) in one year.
58. \( P = \$7884.39 \)
59. \( P = \$7,836,032.85 \)

**Section 1.4.1**

60. \( S = \$11,056.83 \)
61. \( P = \$10,419.96 \)
62. Repayment amount is \( \$4936.16 \).
63. Repayment amount is \( \$4990.40 \).
64. Repayment amount is \( \$3589.64 \).
65. Better to pay \( \$3200 \) in 6 months, of course. The cash savings is \( \$423.08 \).
66. He should pay \( \$2000 \) today.
67. \( P = \$46,728.97 \)
68. Should pay \( \$27,500 \) now.
69. Repayment amount is \( \$12,920.72 \).
70. Each deposit should be \( \$379,247.34 \).

**Section 1.5**
71. \( P = \$2989.37 \)

72. \( P = \$1017.04 \)

73. (a) \$4053.35
(b) \$53.35
(c) Desperation

74. (a) 184 days
(b) \$40,000
(c) 62 days
(d) \$39,322.77
(e) \$677.23

75. \( r = 6.31\% \)

**Section 1.5.1**

76. \( P = \$1485.15 \)

77. \$1511.14

78. \$14,327.41

79. \$14,685.59

80. (a) \$5443.75
(b) \$56.25
(c) Desperation

81. \( r = 37.68\% \)

**Section 1.6**

82. \$1661.64

83. \$4830.92

84. \$268.29

85. \$268.90

86. \$2659.75

87. First payment is \$3095.24 and the second is \$4642.86

**Section 1.6.1**
88. (a) $1738.12  
(b) $1743.70  
(c) $1741.00  
89. $4887.00  
90. $3409.12  
91. $1223.57  
92. $179.65  
93. $264.71  
94. $2763.98  
95. Answers will vary depending on today’s date.  
96. $3,719,436.54  

Section 1.7  
97. (a) $694.24  
(b) No credit card company would let you miss payments without charging you a late fee and/or increasing the interest rate.  
(c) $1594.24  
98. (a) $3103.92  
(b) $6103.92  
(c) $103.92  
99. $472.41  

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<th>Month</th>
<th>Interest Charged/ Paid</th>
<th>Principal Paid</th>
<th>Amount Owed After Payment</th>
</tr>
</thead>
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<td>198.88</td>
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<tr>
<td>2</td>
<td>111.01</td>
<td>200.07</td>
<td>14,601.05</td>
</tr>
</tbody>
</table>

Section 1.7.1  
100. $754.72  
101. $341.94  
102. $480.07  
103. $1577.34
105.  

<table>
<thead>
<tr>
<th>Month</th>
<th>Interest Charged/ Principal</th>
<th>Amount Owed After Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>$132.21</td>
</tr>
<tr>
<td>2</td>
<td>$399.34</td>
<td>$132.90</td>
</tr>
</tbody>
</table>

106. $761.19

Chapter 2

Section 2.1

107. $S = 146.41$

108. (a) $S = 147.75$
     (b) $S = 148.45$
     (c) $S = 148.94$
     (d) $S = 149.13$
     (e) $S = 149.17$

109. (a) $P = 1000, i = \frac{.06}{2}, and n = 5$
     (b) $S = 1159.27$

110. $S = 12,461.82$

111. $S = 476.45$ so you cannot withdraw $500.

Section 2.1.1

112. Either way the answer is $S = 944.78$

113. (a) $S = 2343.32$
     (b) $S = 2345.78$
     (c) $S = 2346.98$

114. $S = 72,999.17$

115. $S = 178,338.96$

116. $S = 475$

117. $S = 611.48$, so the difference is $136.48.$

118. $S = 482.09$, so you cannot withdraw $500.$

119. $S = 546.36$, so after you pay $400, you owe $146.36.$

Section 2.2

120. $1148.50$

121. $2881.32$

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122. $5059.47
123. $10,266.14
124. $5446.83
125. $6680.58
126. $7100.59

Section 2.2.1
127. $2731.17
128. $42,618.53
129. $2934.12
130. $7884.93
131. $3634.60
132. $3496.04

Section 2.3
133. (a) $551.91
   (b) $51.91
   (c) 10.31%
134. 9.381%
135. 8.3%
136. 12%(12)
137. 8.1%(2)
138. $708.07
139. 7.05%(2)

Section 2.3.1
140. (a) 8.16%
    (b) 8.24%
    (c) 8.29%
    (d) 8.33%
141. (a) 12.36%
(b) 12.55%
(c) 12.68%
(d) 12.74%

142. 7.2%(1)
143. 7.2%(2)
144. $15.66
145. $1.01
146. 9.1%(2) is better - it saves him $0.19
147. 8%(12) is better - it saves him $594.36

**Section 2.4**

148. (a) 2
    (b) 2
    (c) $x$
    (d) 2
149. $x \approx 1.26$
150. $x \approx 11.90$
151. $x \approx 37.17$

**Section 2.5**

152. $n \approx 11.8956$ years
153. $n \approx 37.167$ quarters
154. $n \approx 43.89$ quarters
    (a) 44 quarters
    (b) 44 quarters
    (c) 43 quarters and 80 days
155. $n \approx 5.42$ years
    (a) 5 years
    (b) 6 years
    (c) 5 years and 149 days

**Section 2.5.1**

156. $n \approx 23.199$ months
(a) 23 months
(b) 24 months
(c) 23 months and 6 days

157. \( n \approx 132.38 \) months
   (a) 132 months
   (b) 133 months
   (c) 132 months and 11 days

158. \( n \approx 86.45 \) months
   (a) 86 months
   (b) 87 months
   (c) 86 months and 13 days

159. \( n \approx 78.868 \) months, so 79 months.

160. \( n \approx 23.36 \) semiannual periods. Therefore, 23 semiannual periods plus 64 days.

Section 2.6

161. $20,415.49
162. $30.65
163. $7769.44
164. $118,567.01
165. \( NPV = -66.67 \)

166. Not a good investment because you lose money.

Section 2.6.1

167. (a) $24,709.21
    (b) $33,920.46
    (c) $46,565.51

168. $21.34
169. $6554.56
170. $153,531.25
171. $4288.67
172. The note is sold for $4198.31

**Section 2.7**

173. nominal rate is 13.59% and rate per month is 1.13%
174. nominal rate is 23.79% and rate per period is 5.95%
175. 22.47%
176. r=10.65% and S=$3,150,789,721
177. Not reasonable to extrapolate to so far in the future.

**Section 2.7.1**

178. nominal rate is 23.13% and rate per month is 1.92%
179. nominal rate is 23.50% and rate per quarter is 5.89%
180. nominal rate and rate per year are both 25.74%
181. The more often money is compounded, the smaller the nominal rate needs to be.
182. rate per quarter is 6.05% and nominal rate is 24.20%
183. \( r = 5.76\% \) and \( S = 61,274.88 \). This is not reasonable because that is triple the 1990 budget.

**Section 2.8**

184. (a) $11,025
   (b) $11,038.13
   (c) $11,047.13
   (d) $11,050.65
   (e) $11,051.56
   (f) $11,051.56
   (g) $11,051.70
   (h) $11,051.70
   (i) $11,051.71

185. $780.61
186. $340.86
187. 6.93 years
188. 13.12%
189. (a) $260,202.69  
    (b) $252,318.37  
    (c) 4.8%

190. $115,566.62

191. 6.18%

Section 2.8.1

192. 8%\((\infty)\) because it is the same rate, but compounded more often.

193. $258,165.23

194. $252,554.63

195. $2717.26

196. $5637.48

197. $152,292.51

198. 8.33%

199. 12.75%

Chapter 3

Section 3.1

200. $4310.13

201. (a) $16,140.01  
    (b) $15,000  
    (c) $1140.01

202. $1841.10

203. (a) 14  
    (b) $3919.73

204. $3726.75

205. $4420.45

206. For 203, there was $2800 deposited, so $1119.73 earned in interest.  
    For 204, $2650 was deposited, and $1076.75 was earned in interest.  
    For 205, $3140 was deposited and $1280.45 was earned in interest.
207. $126.18

**Section 3.1.1**

208. Either way it should be $506.16
209. $7707.89
210. $18,521.19
211. $2512.90
212. $2227.49
213. Rod because his parents deposit $1500 into his account, while Todd only has $1425 deposited into his.

214. (a) $30,324.28
    (b) $16,000
    (c) $14,324.28
    (d) $141,340.17

**Section 3.2**

215. (a) $10,630.93
    (b) $11,600
    (c) $8625.14+$2000=$10,625.14
    (d) $974.86

216. (a) $232,810.30
    (b) $217,189.70
    (c) $252,810.30
    (d) $333,583.23, so yes

217. The present value of the payments is $19,542.83, so you should make payments.

218. (a) $288,444.68
    (b) $488,444.68
    (c) $148,097.15
    (d) Noted!

219. (a) $40,764.97
    (b) $17,646.09

**Section 3.2.1**
220. (a) $75,762.65
(b) $105,762.65
(c) Nope, he’ll only have $111,664.62.

221. The cash equivalent of the other payments is $9774.29, so use the payment plan.

222. The cash equivalent of the payments is $464.49, so use the payment plan.

223. (a) $109,723.86
(b) $119,723.86

224. (a) $99,800.97
(b) $124,800.97
(c) $31,465.67

225. (a) $113,234.90
(b) $153,234.90
(c) $61,732.25

226. The shorter term because you pay less overall.

Section 3.3

227. $523.81

228. (a) $9151.32
(b) R=$8306.89, so he saves $5066.58

229. $138.11

230. $93.03

231. (a) $295.50
(b) $17,730
(c) $1092.04

Section 3.3.1

232. (a) $250.94
(b) $18067.68
(c) $1429.72

233. $150.83

234. (a) $499.59
(b) $1487.11
235. $46.33

236. (a) $1022.15
    (b) $390.58
    (c) $9648.43

**Section 3.4**

237. (a) $8642.91
    (b) $862.40 per payment

238. $779.91

239. (a) $216.01
    (b) $3782.51

240. (a) $398.72
    (b) $436.22

241. \( R = $155.20 \)

242. This is a bonus problem! Try using Excel to solve it!

**Section 3.5**

243. \( n \approx 13.72 \)
    (a) 14
    (b) 14
    (c) 13 full deposits plus $174.10

244. \( n \approx 10.72 \)
    (a) 11
    (b) 11
    (c) 10 full payments plus $358.76

245. 38

246. $647.06

**Section 3.5.1**

247. (a) 29
    (b) 30
    (c) Extra $67.34

248. (a) 25
    (b) 25
249. (a) 228
(b) 508
(c) $313.46

250. (a) 82
(b) $1667.28

251. (a) 145 months
(b) $207.48

252. 374 months

Section 3.6

253. \( r \approx 29.9\% \)

254. \( r \approx 5.08\% \)

255. \( r \approx 25.15\% \)

Section 3.6.1

256. \( r \approx 4.2\% \)

257. \( r \approx 4.25\% \)

258. \( r \approx 3\% \)

259. \( r \approx 13.09\% \)

260. \( r \approx 18.1\% \)

261. His old account will be worth $17,618.80, so he needs another $17381.20. Depositing $1500 per quarter for 3 years gives him an additional $18,000, so he does not need the new account to pay interest at all to reach his goal.

Section 3.7

262. (a) $23,000
(b) $677.01
(c) $24,372.36
(d) $1372.36
### Section 3.7.1

263. (a) $16,500  
    (b) $293.92  
    (c) $17,635.20  
    (d) $1135.20  
    (e) $248.85  
    (f) $17,917.20  
    (g) $1417.20  
    (h) $2942.19  

264. (a) $936.29  
    (b) $48,899.19  
    (c) $630  
    (d) $306.29  
    (e) $224,709.60  
    (f) $89,709.60  

265. (a) $362.73  
    (b) $131.25  
    (c) $231.48  
    (d) $34,768.52  
    (e) $130.38  
    (f) $232.35  
    (g) $43,527.60  

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount Owed</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$23,072.83</td>
<td>$72.83</td>
<td>$604.18</td>
<td>$22,395.82</td>
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<tr>
<td>2</td>
<td>$22,466.74</td>
<td>$70.92</td>
<td>$606.09</td>
<td>$21,789.73</td>
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<td>3</td>
<td>$21,858.73</td>
<td>$69.00</td>
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<td>4</td>
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<td>$16,299.27</td>
<td>$51.45</td>
<td>$625.56</td>
<td>$15,622.26</td>
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</tbody>
</table>
(h) $8527.60
(i) $19,456.61
(j) $12,193.87

266. (a) $494.43
(b) $145.83
(c) $348.60
(d) $49,651.40
(e) $144.82
(f) $350.11
(g) $59,331.60
(h) $9331.60
(i) $27,178.81
(j) $16,890.61

267. (a) $289.98
(b) $145.83
(c) $144.15
(d) $49,855.85
(e) $145.41
(f) $144.57
(g) $69,595.20
(h) $19,595.20
(i) $40,563.31
(j) $36,302.02

268. (a) $9151.32
(b) $54,907.92
(c) $9907.92
Month | Amount Owed | Interest Paid | Principal Paid | Balance  
--- | --- | --- | --- | ---  
1 | $47,700.00 | $2700 | $6451.32 | $38,548.68  
2 | $40,861.60 | $2312.92 | $6838.40 | $31,710.28  
3 | $33,612.90 | $1902.62 | $7248.70 | $24,461.58  
4 | $25,929.27 | $1467.69 | $7863.63 | $16,777.95  
5 | $17,784.63 | $1006.68 | $8144.64 | $8633.31  
6 | $9151.31 | $518.00 | $8633.32 | -$0.01  

269.  

(a) $63,019.26  
(b) $56,980.74  
(c) $36,275.28  

Month | Amount Owed | Interest Paid | Principal Paid | Balance  
--- | --- | --- | --- | ---  
1 | $63,402.63 | $383.37 | $116.63 | $62,902.63  
2 | $63,285.28 | $382.66 | $117.34 | $62,785.28  
3 | $63,167.23 | $381.94 | $118.06 | $62,667.23  
4 | $63,048.45 | $381.23 | $118.77 | $62,548.45  
5 | $62,928.96 | $380.50 | $119.50 | $62,428.96  
6 | $62,808.73 | $379.78 | $120.22 | $62,308.73  

(e)  

Month | Amount Owed | Interest Paid | Principal Paid | Balance  
--- | --- | --- | --- | ---  
14 | $61,820.24 | $373.80 | $126.20 | $61,320.23